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Geometry builds understanding of geometric concepts and critical skills in spatial reasoning. Students progress from concrete to abstract thinking using algebraic connections and prior knowledge to master core geometry topics.

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Carnegie Learning, Inc.
Pittsburgh, Pennsylvania
1-888-244-7569

www.carnegielearning.com
Foreword

Carnegie Learning strives to incorporate continual improvements into our products and services. Our Curriculum Development Team analyzed the National Council of Teachers of Mathematics (NCTM) and Achieve standards and incorporated their key mathematical concepts into this new edition of our Carnegie Learning® Geometry textbook. We also considered educational research, National Mathematics Standards, best practices, and customer feedback when developing new content and pedagogical approaches.

The van Hiele model is a theory that describes how students learn geometry. Two Dutch educators, Dina van Hiele-Geldof and Pierre van Hiele, created a research-based framework for designing a geometry curriculum. Their theory describes five levels of geometric understanding that in general relate to student experiences rather than age. The van Hieles postulated that students must pass through these levels of understanding to achieve mastery in a content area. Research suggests that most students enter a high school geometry course thinking at Level 1 (Visualization) or Level 2 (Analysis).

<table>
<thead>
<tr>
<th>The van Hiele Model of Geometric Thought¹</th>
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</thead>
<tbody>
<tr>
<td>Level 1</td>
</tr>
<tr>
<td>Level 2</td>
</tr>
<tr>
<td>Level 3</td>
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<tr>
<td>Level 4</td>
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<tr>
<td>Level 5</td>
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</tbody>
</table>

The van Hieles also proposed five sequential phases for guiding students from one level to the next on a given mathematical concept. These learning phases are in line with the Learning by Doing® methodology Carnegie Learning® incorporates into all of its mathematics instruction, and our Curriculum Development Team integrated them into the development of this Geometry textbook. We believe that students achieve mastery when they are presented with quality tasks that engage them in the mathematics. Table 2 demonstrates our correlation to the van Hieles’ phases of learning.
## Table 2

<table>
<thead>
<tr>
<th>van Hieles’ Phases of Learning²</th>
<th>Carnegie Learning Geometry Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information</td>
<td>• Introduction problems are presented for each chapter to assist teacher in assessing students’ knowledge of the key concepts of the chapter.</td>
</tr>
<tr>
<td></td>
<td>• Icons throughout the student text support communication: discuss to understand, think by yourself, work in pairs, work in groups, share solutions and methods with the class.</td>
</tr>
<tr>
<td>Guided Orientation</td>
<td>• Students draw, sketch, construct, and measure. Through this exploration students build an intuitive understanding of properties and characteristics of geometric figures.</td>
</tr>
<tr>
<td></td>
<td>• Questions guide student exploration.</td>
</tr>
<tr>
<td>Explication</td>
<td>• Students make conjectures from their explorations, prove each conjecture, and state formally as theorems.</td>
</tr>
<tr>
<td></td>
<td>• Key terms are introduced when relevant.</td>
</tr>
<tr>
<td></td>
<td>• Students write in a consumable book and explain their reasoning throughout.</td>
</tr>
<tr>
<td>Free Orientation</td>
<td>• Students use definitions, postulates, and theorems to prove additional theorems, just like Euclid.</td>
</tr>
<tr>
<td></td>
<td>• Students solve application problems.</td>
</tr>
<tr>
<td>Integration</td>
<td>• Topics are connected; new knowledge is constructed from prior knowledge gained throughout the text.</td>
</tr>
<tr>
<td></td>
<td>• Students share solutions in pairs, in groups, and with the whole class.</td>
</tr>
</tbody>
</table>

Throughout the Geometry text, we have incorporated common threads: construction, proof, transformation, algebraic reasoning, and composition. These concepts are not presented in isolation but rather revisited within each chapter to strengthen student understanding.

The text provides student-centered tasks with examples and illustrations. Questions that facilitate understanding are embedded in each lesson. If a question is worth asking to foster understanding, it appears in the student text, and not simply in the Teacher’s Implementation Guide.


2 Crowley 5–6.
### Sampler Contents

**Note**
This sampler contains a selection of lessons from Carnegie Learning’s Geometry student and teacher texts. The following partial chapters are provided for each of the text elements:

<table>
<thead>
<tr>
<th>Student Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3 Special Angles</td>
</tr>
<tr>
<td>1.6 Forms of Proof</td>
</tr>
<tr>
<td>Introduction Problem for Chapter 3</td>
</tr>
<tr>
<td>5.6 Indirect Measurement</td>
</tr>
<tr>
<td>6.6 Direct Proof vs. Indirect Proof</td>
</tr>
<tr>
<td>8.4 Decomposing Polygons</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher’s Implementation Guide</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.4 Decomposing Polygons</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher’s Resources and Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignments 8.4</td>
</tr>
<tr>
<td>Skills Practice 8.4</td>
</tr>
<tr>
<td>Chapter 3 Assessments</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Collaborative Classroom</th>
</tr>
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<tbody>
<tr>
<td>8</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Geometry Table of Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
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</table>

<table>
<thead>
<tr>
<th>Geometry Text Alignment with National Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
</tr>
</tbody>
</table>

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Sampler Contents

Student Text Set............................... p. 29

The Geometry Student Text is a consumable textbook designed for students to take notes and work problems directly in each lesson. Each lesson contains Objectives, Key Terms, and problems that help the students to discover and master mathematical concepts. The Student Text Set also includes a Student Assignments and Skills Practice book, which contains one assignment and one skills practice per lesson. The Student Assignments and Skills Practice book is designed to move with the student from classroom to home to lab time so that students can continually practice the skills taught in the lesson.

Teacher’s Implementation Guide..................... p. 91

The Geometry Teacher’s Implementation Guide contains a lesson map for each student text lesson. The lesson map includes each lesson’s Objectives, Key Terms, National Standards, Essential Questions, Warm Up Questions, Motivator, and Follow Up. An image of each student text page, including answers, is provided in the Teacher’s Implementation Guide.

Teacher’s Resources and Assessments..................... p. 105

The Geometry Teacher’s Resources and Assessments contains five tests per chapter of the student text. The tests are a Pre-test, a Post-test, a Mid-Chapter Test, an End-of-Chapter test, and a Standardized Test Practice. The Teacher’s Resources and Assessments book contains the assessments with answers in place and also includes the Student Assignments and Skills Practice with answers.

The Carnegie Learning® Test Generator powered by ExamView® Assessment Suite, which includes Assessment, Assignment, and Skills Practice bank files, is available for purchase.
Author’s Note

Students enter geometry classes with varying degrees of experience and mathematical success. New knowledge is built on prior knowledge, which is recognized, validated, and occasionally reshaped. Prior knowledge that is fragmented or based on memorization rather than understanding is an unstable basis for developing new mathematical relationships and concepts. Assimilation, the process of relating new information to already existing cognitive structures, and accommodation, the process of modifying existing cognitive structures so that the new information may be assimilated, are important processes for student learning. This text uses both assimilation and accommodation to integrate knowledge and understanding.

Assessing students’ prior knowledge can be a daunting task. If we do not ask the right questions, we could miss learning opportunities to correct misinformation or spend time on concepts students already understand. To help you assess prior knowledge, each chapter in this text begins with a task designed to reflect a key concept of the chapter. The focus of these tasks should not be only about the correct answers, but also on the strategies students use, for these strategies reveal underlying thought processes. Analyzing processes uncovers students’ strengths and weaknesses, and reveals prior knowledge that needs to be reinforced, de-emphasized, reshaped, or developed.

Chapter 1 begins with an introduction to the undefined terms—point, line, and plane. With this knowledge, students embark on a journey toward establishing a system of Euclidean geometry. From the undefined terms, postulates are introduced, additional vocabulary evolves, and students begin to make their own conjectures. Conjectures become theorems only when students have proven them. Students create many forms of proof throughout the course: flowchart proof, two-column proof, paragraph proof, proof by construction, and indirect proof.

The textbook was intentionally designed to establish geometric connections through the continual process of making conjectures, establishing new vocabulary, and creating proofs. Students will be empowered to construct their own knowledge and engage in the mathematics. The text is filled with “AHA!” moments where connections between different concepts are revealed. In addition to the text, the Cognitive Tutor Geometry software provides students with a rich set of applications.

Rather than lecturing, the teacher guides and supervises as students build the Euclidean system of geometry and learn by doing. The Teacher’s Implementation Guide provides notes, essential ideas, and questions to ensure that the objectives of the lessons are achieved.

Students working through this text will have opportunities to make the connections that lead to a deeper understanding of geometry.

Jaclyn Snyder
As you begin the process of planning for the school year, you will want to give serious consideration to how your classroom is structured. Early research on teaching and learning has revealed that what happens in the classroom in the first three days determines the environment for the entire year. This insight is important as you begin to think about your classroom and the Cognitive Tutor® Geometry curriculum. An effective implementation of the curriculum is most likely to occur in the collaborative classroom, a classroom in which knowledge is shared.

Carnegie Learning’s philosophy—Learning by Doing®—captures the belief that students develop understanding and skill by taking an active role in their environment. Furthermore, it is Carnegie Learning’s belief that effective communication and collaboration are essential skills for the successful learner. It is through dialogue and discussion of different strategies and perspectives that students become knowledgeable independent learners. These beliefs can be realized in the collaborative classroom.

Defining a Collaborative Classroom

A collaborative classroom is an environment in which knowledge and authority are shared between the teacher and the students. In a collaborative classroom, teachers are facilitators and students are active participants. All students, not segregated by ability level, interest, or achievement, benefit from the environment created in the collaborative classroom.

Teachers in the collaborative classroom combine their extensive knowledge about teaching and learning, content, and skills with the informal and formal knowledge, strategies, and individual experiences of their students. The collaborative classroom differs from the traditional classroom in which the teacher is seen as an information giver (Tinzmann, M.B.; Jones, B.F.; Fennimore, T.F.; Bakker, J; Fine, C.; and Pierce, J., 1990).

Characteristics of the Collaborative Classroom

The collaborative classroom is identified by discussion, with in-depth accountable talk and two-way interactions, whether among members of the whole class or small groups. It is a well-structured environment in which questioning and dialogue are valued and appropriate parameters are set so that active learning can occur. Careful planning by the teacher ensures that students can work together to attain individual and collective goals and to develop learning strategies.

In the collaborative classroom, students are encouraged to take responsibility for their learning through monitoring and reflective self-evaluation. The collaborative classroom is one in which teachers spend more time in true academic interactions as they guide students to search for information and help students to share what they know. As facilitators, teachers have the opportunity to provide the correct amount of help to individual students by providing appropriate hints, probing questions, feedback, and help in clarifying thinking or the use of a particular strategy.
Collaborative Learning versus Cooperative Learning

Two types of learning occur in the collaborative classroom: collaborative learning, which focuses on interaction, and cooperative learning, which is a structure of interaction that helps students to accomplish a goal or end product. While these two forms of learning are often described and used interchangeably, differences do exist. The significant difference between collaborative and cooperative learning environments is the amount of control that the teacher exercises in setting goals and providing choice. For instance, in a collaborative classroom, students are positioned to set their own goals and choose activities, whereas in the cooperative learning environment, the teacher directs these activities.

Learning in the Collaborative Classroom

Critical to teaching and learning in the collaborative-cooperative environment is the ability to define the responsibilities of the teacher and students. For effective collaboration and cooperative teamwork, teachers and students must agree to certain responsibilities that support the learning process. The table shown reflects the parallel responsibilities of teachers and students.

Effective Collaboration and Cooperative Teamwork

<table>
<thead>
<tr>
<th>Teacher Responsibilities</th>
<th>Student Responsibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor student behavior.</td>
<td>Develop the skills to work cooperatively.</td>
</tr>
<tr>
<td>Provide assistance when needed.</td>
<td>Learn to talk and discuss problems with each other in order to accomplish the group goal.</td>
</tr>
<tr>
<td>Answer questions only when they are group questions.</td>
<td>Ask for help only after each person in the group has considered the problem and the group has a question for the teacher.</td>
</tr>
<tr>
<td>Interrupt the process to reinforce cooperative skill or to provide direct instructions to all students.</td>
<td>Believe that all members of the group work together toward a common goal. Understand that the success or failure of the group is to be shared by all members.</td>
</tr>
<tr>
<td>Provide closure for the lesson.</td>
<td>Reflect on the work of the group.</td>
</tr>
<tr>
<td>Evaluate the group process by discussing the actions of the group members.</td>
<td>Appreciate that working together is a process and encourage each group member to interact and relate to the rest of the group members.</td>
</tr>
<tr>
<td>Help students to become individually accountable for learning and reinforce this understanding regularly.</td>
<td>Realize that each member must contribute as much as he or she can to the group goal. Understand that the success of the group is dependent on the individual work of each member of the group. Understand that group members are individually accountable for their own learning.</td>
</tr>
</tbody>
</table>
What the Collaborative Classroom is Not

It must be agreed upon by the teacher and the students that the collaborative classroom is not one in which students:

- Work in small groups on a problem or group of problems without direction or individual responsibility.
- Work individually while sitting in a group working on problems.
- Work without conversation or interaction regarding the method or process being used to solve the problem.
- Allow one member of the group to do all of the work while others sit passively.

Shaping the Collaborative Classroom

To ensure that the spirit and purpose of the collaborative classroom is clear from the onset of school, you will want to engage your students in a collaborative activity on the first day. In doing so, you can accomplish two important goals. First, students immediately understand the importance and value of working together, and secondly, students quickly move into their role as active participants.

The activity “Facts in Five,” as you may have experienced in training, is an activity designed to meet these goals. Another popular activity with students is known as “Broken Squares” (Spencer Kagan: Cooperative Learning©). In this activity, members of the team are each given several pieces of a broken square. The pieces belong to different squares. Students must create the whole square by taking turns giving each other one piece. No one may speak during the activity; that is, no one can ask for what he or she needs. This activity is perfect for teaching sensitivity and the importance of communication.

During the first few days of class, it is extremely important that expectations and the “rules of the game” be defined. The best approach is to have the students work together in small groups to generate the guidelines for teamwork (See Lesson: Creating Collaborative Classroom Guidelines on page 13).

As a guidepost for identifying the elements for successful group interactions, we suggest reviewing the “Ten guidelines for students doing group work in mathematics” written by Anne E. Brown for the CLUME Project (http://www.uwplatt.edu/clume/tenguide.htm). Brown developed these guidelines after viewing the video and audio tapes of more than a dozen group sessions of her students. This list reflects the apparent actions critical to the success or failure of the group. In summary, the guidelines state the following:

1. Groups should be formed quickly and members of the group should sit together, facing each other, and get to work quickly. Members should call each other by first name. Members should not engage in “off-task” discussion. Everyone should be encouraged to participate.
2. All instructions should be read aloud so that everyone is aware of the expectations of the assignment.
3. Members of the group should listen to each other and not interrupt. Comments or questions should be acknowledged and responded to by other group members.

4. Members of the group should not accept being confused. If a member of the group does not understand the information that is presented, this person should ask someone to paraphrase or rephrase what was said.

5. Members of the group should ask for clarification if a word is used in a way that is confusing.

6. The members of the group should work together on the same problem and check for agreement frequently.

7. Members of the group should explain their reasoning by “thinking out loud” and ask others to do the same. This helps everyone to relate the information being presented to what they already know.

8. Members of the group should monitor the group’s progress and be aware of time constraints so that all members of the group meet the goals of the assignment.

9. If the group gets stuck, the members of the group should review and summarize what they have done so far. The group can then ask for questions to find errors or missing connections to help the group’s work to proceed.

10. Members of the group should engage in questioning, the engine that drives mathematical investigation.

Group Work in the Collaborative Classroom

If we expect students to work well in groups, they will need to understand what it means to learn collaboratively and how it will benefit them. A good description of collaborative learning used by many of our teachers is:

Collaborative learning is a process in which each individual contributes personal knowledge and skill with the intent of improving his or her learning accomplishments along with those of others.

Students should be aware that one of the most important goals of collaborative learning is to create a “community of learners.” They should understand that the community will grow and thrive only if all members of the group are active participants. Students must also understand that their role in the classroom will be different than what they may have experienced in other classes and so will the teacher’s role!

You will want to introduce the features of a collaborative classroom to your students. Important characteristics of the collaborative classroom include:

- Shared responsibility
- Choice
- Discussion about how we learn from what is right as well as what is wrong
- Working in groups, whether as an entire class or as several small groups
Finally, you will want students to understand the goals and expectations of a collaborative classroom:

- Students learn collaboratively to gain greater individual proficiency.
- Groups “sink or swim” together.
- EVERYONE suggests, questions, and encourages.
- Group members are responsible for each other’s learning.
- All group members bring valued talents and information to the task at hand.

**Getting Started in the Collaborative Classroom**

When problems and investigations in the text require that students work in groups, you will want to structure the groups. When problems and investigations in the text require that students work individually, it is possible to maintain a collaborative classroom where students are free to communicate with each other and to share information.

To form groups initially, you may want to set arbitrary groups and make changes as you observe students. One suggestion for structuring groups is to think about having two types of groups: long-term groups and short-term groups. The long-term groups, or home groups, stay together for the entire school year and sit together in class. Long-term groups enable students to build trust and confidence and to learn how to negotiate with each other to derive success. On the other hand, the short-term groups are randomly assigned for specific tasks. Short-term groups allow students to develop the ability to work with many different people. Clearly, how you arrange the groups will depend on how to best meet your students’ needs.

Most importantly, you want to make sure that students are respectful of one another at all times. The success of the group depends on cooperation, which can be achieved only if students accept one another and value the contributions of others.

If you have students who do not want to work in groups, do not force the issue. Allow those students to work alone. It is important that the student who is working alone understands that the teacher is not a member of his or her group. After these students find that they cannot talk with others and that those who are sharing information are progressing more easily, they will naturally gravitate to a group.

You want to structure the success of the group experience, so it is important to use guidelines and timelines. Although you will want the students to come up with the operating guidelines, timelines are probably better left to you to determine.

After the groups are formed, you may want to have one person from each group be designated as a facilitator. Some responsibilities of the facilitator include:

- Obtaining and returning all materials
- Communicating information from the teacher to the group
- Handing in the completed assignments for the group
Success while working collaboratively depends upon every group member working on every part of the problem, so you may find that you do not want to assign roles such as recorder or reader to group members.

Students working together should generate noise and movement in the room. Some have defined this attribute as “controlled chaos.” To ensure that the group work remains in control, you will want to monitor group interactions and check for understanding of the task at hand. You may also want to ask students to complete parts of the problem or investigation, stop and discuss the work done, summarize the main points of the task, and then continue. This works well when the problem or investigation is lengthy.

Because groups will work at different paces, you might want to prepare some additional tasks or extensions of the problem or investigation for those groups who finish quickly.

Note that graphing calculators are assumed to be available to students throughout the course.

Facilitating Groups in the Collaborative Classroom

Facilitating the group process is critical. As noted earlier, you should only answer a question posed by the group rather than by individual students. You may also restrict the number of questions that a group can ask, being generous the first few times that students work in groups. When a group asks questions, answer by redirecting with guiding questions such as:

- What does your group think?
- How did you arrive at that answer?
- How does this relate to past activities?
- What work have you done so far?
- What do you know about the problem?
- What do you need to figure out?
- What materials might help you to figure this out?
- Are there other parts of the problem that you can do first?

Other tips to consider as you manage your collaborative classroom include:

- Provide additional instruction to those struggling with a task.
- Listen carefully and value diversity of thought that often provides instructional opportunities.
- Balance learning with working effectively. Remember that no one is on task 100% of the time.
- Deal with conflict constructively.
- Ask students to sign off on other group members’ papers to acknowledge that everyone understands the group’s results.
Holding the groups accountable for an end product, such as a presentation, will add further value to the learning activity. As you have surely discovered, when you truly understand a concept or idea yourself, then you are able to explain that concept or idea to someone else.

**Presentations and Discussions in the Collaborative Classroom**

To successfully close or wrap up a problem or investigation with a presentation and discussion, students must know exactly what you expect from them. You should also make sure that students know that you will hold the entire group accountable for the presentation. (This helps to ensure that students will hold each other accountable.)

Some suggestions for facilitating the presentation process include:

- Choose presenters in a group to ensure that all students have the opportunity to present.
- Require that students defend and talk about their solutions.
- Hold all students accountable by asking questions of group members who are not presenting.
- Ask presenters to make connections and generalizations and extend concepts.
- Allow groups time to process feedback and to celebrate their achievements.

To bring closure to the group work and presentation process, engage students in discussion or have them keep learning journals. Some suggestions for summary wrap-up questions include:

- What was something that you learned from this problem?
- What were the mathematical concepts that you applied in solving this problem?
- About what concepts do you still have questions?
- What are three things that your group did well?
- What is at least one thing that your group could do even better the next time?

**Conventions for Students’ Written Responses**

When working in the Student Text, students should always try to answer questions with complete sentences. Full sentences should also be encouraged in written answers to Assignment or Assessment questions. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context. In both the Teacher’s Implementation Guide and the Teacher’s Resources and Assessments, answers are provided in the form of complete sentences whenever appropriate.

Note that many answers in the Teacher’s Implementation Guide and Teacher’s Resources and Assessments are samples of correct answers. Students may present correct responses that do not exactly match the sample answers provided in the teacher’s materials. Actual student responses may include alternate solution paths or interpretations of a given question. This variety provides teachers with a rich opportunity for classroom exploration and discussion.
Checklist of Teacher-Directed and Learner-Centered Classrooms

To understand where you are in the transition process from creating a teacher-directed classroom to creating a learner-centered classroom, you may use the criteria shown to evaluate your classroom (courtesy of Jacquelyn Snyder, Jan Sinopoli, and Vince Vernachhio, Pittsburgh Public Schools). Use your initial evaluation as a baseline measure and check yourself at regular intervals throughout the school year.

<table>
<thead>
<tr>
<th>Teacher-Directed Classroom</th>
<th>Learner-Centered Classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher directs all classroom activity.</td>
<td>The teacher facilitates classroom activity.</td>
</tr>
<tr>
<td>Each activity is dependent on the teacher.</td>
<td>Most activities require only guidance from the teacher.</td>
</tr>
<tr>
<td>The teacher is in the front of the room instructing the entire class using</td>
<td>The teacher walks around the classroom during all activities, watching and listening to</td>
</tr>
<tr>
<td>the blackboard or overhead most of the time.</td>
<td>student-to-student discourse.</td>
</tr>
<tr>
<td>The teacher models examples of the lesson objective and directs students</td>
<td>The teacher monitors the students to keep them on task, while the students actively work</td>
</tr>
<tr>
<td>to practice similar problems found in the text or on handouts designed by</td>
<td>together on an activity.</td>
</tr>
<tr>
<td>the teacher.</td>
<td></td>
</tr>
<tr>
<td>Students are seated in rows, working as a class with the teacher at the</td>
<td>The students are typically paired or grouped to work together while the teacher facilitates the process.</td>
</tr>
<tr>
<td>front of the class or working independently.</td>
<td></td>
</tr>
<tr>
<td>The teacher presents the material while students watch and take notes.</td>
<td>The teacher systematically brings the class together on several occasions, assuring that the mathematics of the lesson are understood.</td>
</tr>
<tr>
<td>The students work independently as the teacher tries to help each student</td>
<td>Students are required to make presentations, explaining their progress within the activity.</td>
</tr>
<tr>
<td>The teacher completely answers the problem for the student when he or she</td>
<td>If a student is having difficulty understanding something, even after consulting with his or her group members, the teacher asks the group leading questions to guide them to the desired outcome.</td>
</tr>
<tr>
<td>is having difficulty.</td>
<td></td>
</tr>
<tr>
<td>The teacher does the thinking and the work.</td>
<td>The students do the thinking and the work.</td>
</tr>
<tr>
<td>The teacher asks low level or fill-in-the-blank types of questions that</td>
<td>The teacher asks thought-provoking questions that required students to explain their thinking and processes.</td>
</tr>
<tr>
<td>can be answered with a single number or in a word or two.</td>
<td></td>
</tr>
<tr>
<td>The majority of classroom discourse is teacher-to-student discourse.</td>
<td>The majority of discourse is student-to-student discourse.</td>
</tr>
<tr>
<td>The teacher encourages students to memorize rules, procedures, and</td>
<td>The teacher encourages students to construct knowledge. Prior knowledge is assessed as new concepts emerge.</td>
</tr>
<tr>
<td>formulas.</td>
<td></td>
</tr>
</tbody>
</table>
Lesson: Creating Collaborative Classroom Guidelines

Preparing for the Lesson

- Arrange the class seats in groups of three or four such that students face each other. Position the desks in such a way that students need only do a half turn of their heads if you call their attention to the front of the room.
- Give poster boards to each group.
- Give colorful markers to each group.

Expected Student Growth

- Students will gain experience in working cooperatively, listening, and respecting the ideas of others, and coming to a consensus regarding the final product.
- Students will learn how to share power with the teacher.

Initiating the Activity

- Ask students if they have ever worked in groups.
- Ask students to think about good and not-so-good group experiences.
- Have students make lists of things that happen in groups or things that they think should happen in groups to have a group work more productively to complete a task.
- Direct students to develop social guidelines for group work in class. The guidelines should be phrased positively and refer to observable behavior. Lists of guidelines should not be too long.

Facilitating the Activity

- Monitor student behavior.
- Offer assistance only if necessary. For instance, students may be making their lists too long.
- Interrupt the process to reinforce cooperative skills or to provide directions.

Student Presentations

- Have all groups present their guidelines.
- Have students determine which guidelines are similar and record those.
- Have students look at the remainder of the guidelines and determine which should be included in the list of guidelines. Students should be able to justify their choices. As all students will be using these guidelines, there should be consensus on the final list.

Lesson Closure

- Indicate that the final list will be generated and every member in the class must agree by signing off on the list. By doing so, students have agreed to honor the list of guidelines and will be held accountable.
- Indicate that groups not adhering to the guidelines may have their group grades reduced.
Presentation Rubric

The rubric can be used to help you score group presentations to the entire class. The presentation scores, which range from 1 to 5, are detailed in the rubric. It is a good idea to copy the Presentation Rubric and distribute it to the class so that students understand how they are scored.

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<tr>
<td>5</td>
<td>You earn a 5 for your presentation if your presentation is nearly perfect. Your mathematics must be correct with only a very minor flaw (not having to do with the main idea of the problem). Your public speaking skills must also be perfect or quite close to perfect. You must look at your audience. Your must present yourself well and not make distracting gestures or hand motions during the presentation. Your rate of speech must be neither too fast nor too slow.</td>
</tr>
<tr>
<td>4</td>
<td>You earn a 4 for your presentation if you miss one thing within the mathematical content of your presentation. OR You earn a 4 for your presentation if there is one thing that you do not do very well within the public speaking part of the presentation.</td>
</tr>
<tr>
<td>3</td>
<td>You earn a 3 for your presentation if you can complete the problem, but your public speaking skills are poor. This score means that you do not make eye contact, you speak inaudibly, you mumble your words, etc. OR You earn a 3 for your presentation if you have some content knowledge and make one major error, as well as omit one of the important aspects of good public speaking.</td>
</tr>
<tr>
<td>2</td>
<td>You earn a 2 for your presentation if you stand up for your presentation but really have very little content knowledge. This score means that you are unable to complete the problem and your speaking skills are poor.</td>
</tr>
<tr>
<td>1</td>
<td>You earn a 1 for your presentation for being willing to stand up and try to present.</td>
</tr>
<tr>
<td>0</td>
<td>You earn a 0 for your presentation if you refuse to stand up and try to present.</td>
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A Note on National Standards

In the Learning by Doing Lesson Map that precedes every lesson in this Geometry Teacher’s Implementation Guide, you will find the NCTM Standard(s), which are addressed in the lesson content.
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1 Tools of Geometry
   1.1 Points, Lines, Planes, Rays, and Segments
   1.2 Naming Angles, Classifying Angles, Duplicating Angles, and Bisecting Angles
   1.3 Complements, Supplements, Midpoints, Perpendiculars, and Perpendicular Bisectors
   1.4 Two Methods of Logical Reasoning
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   4.2 Simplifying Radicals, Pythagorean Theorem, and Its Converse
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# Geometry

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Introduction

During this course, you will solve problems and work with many different representations of mathematical concepts, ideas, and processes to better understand the world. The following process icons are placed throughout the text.

Construction
- Use only a compass and a straightedge to create the geometric figure.

Discuss to Understand
- Read the problem carefully.
- What is the context of the problem? Do you understand it?
- What is the question that you are being asked? Does it make sense?

Think for Yourself
- Do I need any additional information to answer the question?
- Is this problem similar to some other problem that I know?
- How can I represent the problem using a picture, a diagram, symbols, or some other representation?

Work with Your Partner
- How did you do the problem?
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to one another?

Work with Your Group
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to one another?
- How can we explain our solution to one another? To the class?

Share with the Class
- Here is our solution and how we solved it.
- We could only get this far with our solution. How can we finish?
- Could we have used a different strategy to solve the problem?
OBJECTIVES
In this lesson you will:
- Calculate the supplement of an angle.
- Calculate the complement of an angle.
- Construct a perpendicular.
- Construct a perpendicular bisector.
- Construct the midpoint of a segment.
- Classify adjacent angles, linear pairs, and vertical angles.

KEY TERMS
- supplementary angles
- complementary angles
- perpendicular
- midpoint of a segment
- segment bisector
- perpendicular bisector
- adjacent angles
- linear pair
- vertical angles

PROBLEM 1  Supplements and Complements

Two angles are **supplementary angles** if the sum of their angle measures is equal to 180°.

1. Use a protractor to draw a pair of supplementary angles that share a common side, and then measure each angle.

2. Use a protractor to draw a pair of supplementary angles that do not share a common side, and then measure each angle.
3. Calculate the measure of an angle that is supplementary to \( \angle K\!\!\!J\!\!\!L \).

Two angles are **complementary angles** if the sum of their angle measures is equal to 90º.

4. Use a protractor to draw a pair of complementary angles that share a common side, and then measure each angle.

5. Use a protractor to draw a pair of complementary angles that do not share a common side, and then measure each angle.

6. Calculate the measure of an angle that is complementary to \( \angle J \).
7. Two angles are both congruent and supplementary. What is the measure of each angle? Explain.

8. Two angles are both congruent and complementary. What is the measure of each angle? Explain.

9. The complement of an angle is twice the measure of the angle. What is the measure of each angle? Explain.

10. The supplement of an angle is half the measure of the angle. What is the measure of each angle? Explain.
Two lines, line segments, or rays are **perpendicular** if they intersect to form 90º angles. The perpendicular symbol is \( \perp \).

1. Name all angles that you know are right angles in the figure shown.

2. Draw \( AB \perp CD \) at point \( E \). How many right angles are formed?

3. Draw \( BC \perp AB \) at point \( B \). How many right angles are formed?
1. Construct a line perpendicular to the given line through point $P$. 

- **Draw an Arc**: Use $B$ as the center and draw an arc. Label the intersections points $C$ and $D$.
- **Draw Arcs**: Open the compass radius. Use $C$ and $D$ as centers and draw arcs above and below the line. Label the intersections points $E$ and $F$.
- **Draw a Line**: Use a straightedge to connect points $E$ and $F$. Line $EF$ is perpendicular to line $CD$. 
Perpendicular Line Through a Point Not on a Line

2. Construct a line perpendicular to $\overrightarrow{AG}$ through point $B$.

3. How is the construction of a perpendicular through a point on the line different than the construction of a perpendicular through a point not on the line?
The midpoint of a segment is a point that divides the segment into two congruent segments, or two segments of equal measure.

\[ PMQ \]
\[ P \quad M \quad Q \]

\( PQ \) has midpoint \( M \).

A segment bisector is a line, line segment, or ray that divides the line segment into two line segments of equal measure, or two congruent line segments.

A perpendicular bisector is a line, line segment, or ray that intersects the midpoint of a line segment at a 90 degree angle.

**Perpendicular Bisector**

1. **Draw an Arc**
   - Open the radius of the compass to more than half the length of line segment \( AB \).
   - Use endpoint \( A \) as the center and draw an arc.

2. **Draw an Arc**
   - Keep the compass radius and use point \( B \) as the center and draw an arc.
   - Label the points formed by the intersection of the arcs point \( E \) and point \( F \).

3. **Draw a Line**
   - Connect points \( E \) and \( F \).
   - Line segment \( EF \) is the perpendicular bisector of line segment \( AB \).
1. Construct the perpendicular bisector of $FG$. Label the perpendicular bisector as $\overline{CD}$.

2. Label the point at which $\overline{CD}$ intersects $FG$ as point $E$.

3. If $\overline{CD} \perp \overline{FG}$, what can you conclude?

4. If $\overline{CD}$ bisects $FG$, what can you conclude?

5. If $\overline{CD}$ is the perpendicular bisector of $FG$, what can you conclude?

6. Construct the midpoint of $PQ$. 
PROBLEM 5

Adjacent Angles

∠1 and ∠2 are adjacent angles.

∠1 and ∠2 are not adjacent angles.

1. Describe adjacent angles.

2. Draw ∠2 so that it is adjacent to ∠1.

3. Is it possible to draw two angles that share a common vertex but do not share a common side? If so, draw an example. If not, explain.
4. Is it possible to draw two angles that share a common side, but do not share a common vertex? If so, draw an example. If not, explain.

Adjacent angles are two angles that share a common vertex and share a common side.

PROBLEM 6 Linear Pairs

1. Describe a linear pair of angles.

2. Draw \( \angle 2 \) so that it forms a linear pair with \( \angle 1 \).

3. Name all linear pairs in the figure shown.
4. If the angles that form a linear pair are congruent, what can you conclude?

A linear pair of angles are two adjacent angles that have noncommon sides that form a line.

**PROBLEM 7**  
**Vertical Angles**

∠1 and ∠2 are vertical angles.  
∠1 and ∠2 are not vertical angles.

1. Describe vertical angles.

2. Draw ∠2 so that it forms a vertical angle with ∠1.
3. Name all vertical angle pairs in the diagram shown.

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

4. Measure each angle in Question 3. What do you notice?

Be prepared to share solutions and methods.
OBJECTIVES
In this lesson you will:
• Use the addition and subtraction properties of equality.
• Use the reflexive, substitution, and transitive properties.
• Write a paragraph proof.
• Complete a two-column proof.
• Perform a construction proof.
• Complete a flow chart proof.

PROBLEM 1
Properties of Real Numbers in Geometry

Many properties of real numbers can be applied in geometry. These properties are important when making conjectures and proving new theorems.

The Addition Property of Equality states: “If a, b, and c are real numbers and a = b, then a + c = b + c.”

The Addition Property of Equality can be applied to angle measures, segment measures, and distances.

• Angle measures: If \( m\angle 1 = m\angle 2 \), then \( m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3 \).
• Segment measures: If \( m\overline{AB} = m\overline{CD} \), then \( m\overline{AB} + m\overline{EF} = m\overline{CD} + m\overline{EF} \).
• Distances: If \( AB = CD \), then \( AB + EF = CD + EF \).

1. Write a statement that applies the Addition Property of Equality to angles.

2. Write a statement that applies the Addition Property of Equality to segments.
The **Subtraction Property of Equality** states: “If \( a, b, \) and \( c \) are real numbers and \( a = b \), then \( a - c = b - c \).”

The Subtraction Property of Equality can be applied to angle measures, segment measures, and distances.

- **Angle measures:** If \( m\angle 1 = m\angle 2 \), then \( m\angle 1 - m\angle 3 = m\angle 2 - m\angle 3 \).
- **Segment measures:** If \( m\bar{AB} = m\bar{CD} \), then \( m\bar{AB} - m\bar{EF} = m\bar{CD} - m\bar{EF} \).
- **Distances:** If \( AB = CD \), then \( AB - EF = CD - EF \).

3. Write a statement that applies the Subtraction Property of Equality to angles.

4. Write a statement that applies the Subtraction Property of Equality to segments.

The **Reflexive Property** states: “If \( a \) is a real number, then \( a = a \).”

The Reflexive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

- **Angle measures:** \( m\angle 1 = m\angle 1 \)
- **Segment measures:** \( m\bar{AB} = m\bar{AB} \)
- **Distances:** \( AB = AB \)
- **Congruent angles:** \( \angle 1 \equiv \angle 1 \)
- **Congruent segments:** \( \bar{AB} \equiv \bar{AB} \)

5. Write a statement that applies the Reflexive Property to angles.

6. Write a statement that applies the Reflexive Property to segments.

The **Substitution Property** states: “If \( a \) and \( b \) are real numbers and \( a = b \), then \( a \) can be substituted for \( b \).”

The Substitution Property can be applied to angle measures, segment measures, and distances.

- **Angle measures:** If \( m\angle 1 = 56^\circ \) and \( m\angle 2 = 56^\circ \), then \( m\angle 1 = m\angle 2 \).
- **Segment measures:** If \( m\bar{AB} = 4 \text{ mm} \) and \( m\bar{CD} = 4 \text{ mm} \), then \( m\bar{AB} = m\bar{CD} \).
- **Distances:** If \( AB = 12 \text{ ft} \) and \( CD = 12 \text{ ft} \), then \( AB = CD \).
7. Write a statement that applies the Substitution Property to angles.

8. Write a statement that applies the Substitution Property to segments.

The **Transitive Property** states: “If \(a, b,\) and \(c\) are real numbers, \(a = b,\) and \(b = c,\) then \(a = c.\)"

The Transitive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

- **Angle measures:** If \(m \angle 1 = m \angle 2\) and \(m \angle 2 = m \angle 3,\) then \(m \angle 1 = m \angle 3.\)
- **Segment measures:** If \(m \overline{AB} = m \overline{CD}\) and \(m \overline{CD} = m \overline{EF},\) then \(m \overline{AB} = m \overline{EF}.\)
- **Distances:** If \(AB = CD\) and \(CD = EF,\) then \(AB = EF.\)
- **Congruent angles:** If \(\angle 1 \equiv \angle 2\) and \(\angle 2 \equiv \angle 3,\) then \(\angle 1 \equiv \angle 3.\)
- **Congruent segments:** If \(\overline{AB} \equiv \overline{CD}\) and \(\overline{CD} \equiv \overline{EF},\) then \(\overline{AB} \equiv \overline{EF}.\)

9. Write a statement that applies the Transitive Property to angles.

10. Write a statement that applies the Transitive Property to congruent segments.

**PROBLEM 2**  
Various Forms of Proof

A conditional statement is true if it can be proven to be true. A proof can be presented in many different forms, including:

- A **paragraph proof** is a proof that the steps and corresponding reasons are written in complete sentences.
- A **two-column proof** is a proof in which the steps are written in the left column and the corresponding reasons in the right column. Each step is numbered and the same number is used for the corresponding reason.
- A **construction proof** is a proof that results from creating an object with specific properties using only a compass and straightedge.
- A **flow chart proof** is a proof in which the steps and corresponding reasons are written in boxes. Arrows connect the boxes and indicate how each step and reason is generated from one or more other steps and reasons.
In this section, you will use each of these forms of proof. You may find some forms of proof to be easier or shorter but you should be familiar with all four forms.

1. Draw four collinear points. Label the points $A$, $B$, $C$, and $D$ such that point $B$ lies between points $A$ and $C$, point $C$ lies between points $B$ and $D$, and $AB \equiv CD$.

2. Consider the conditional statement: If $AB \equiv CD$, then $AC \equiv BD$. Write the hypothesis as the “Given” and the conclusion as the “Prove.”
   Given:
   Prove:

3. Complete the flow chart proof of the conditional statement in Question 2 by writing the reason for each statement in the boxes provided.
   Given:
   Prove:
4. Create a two-column proof of the conditional statement in Question 2. Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

**Given:**

**Prove:**

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5. Write a paragraph proof of the conditional statement in Question 2. Each row of the two-column proof in Question 4 should appear as a sentence in the paragraph proof.

6. Create a proof by construction of the conditional statement in Question 2.

```
A  B  C  D
```

**Given:**

**Prove:**
PROBLEM 3  Proof of the Right Angle Congruence Theorem

The Right Angle Congruence Theorem states: "All right angles are congruent."

Given: \( \angle ACD \) and \( \angle BCD \) are right angles.

Prove: \( \angle ACD \equiv \angle BCD \)

Complete the flow chart of the Right Angle Congruence Theorem by writing the statement for each reason in the boxes provided.

- Given
- Definition of right angles
- Transitive Property of Equality
- Definition of congruent angles
The Congruent Supplement Theorem states: “If two angles are supplements of the same angle or of congruent angles, then the angles are congruent.”

1. Use the diagram to write the “Given” statements for the Congruent Supplement Theorem. The “Prove” statement is provided.

Given:
- \( \angle 1 \) is supplementary to \( \angle 2 \)
- \( \angle 3 \) is supplementary to \( \angle 4 \)

Prove: \( \angle 1 \equiv \angle 3 \)

2. Complete a flow chart proof of the Congruent Supplement Theorem by drawing arrows to connect the steps in a logical sequence.

- \( m\angle 1 + m\angle 2 = 180^\circ \)
  - Definition of supplementary angles
- \( m\angle 3 + m\angle 4 = 180^\circ \)
  - Definition of supplementary angles
- \( m\angle 2 + m\angle 4 \)
  - Definition of congruent angles
- \( m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4 \)
  - Substitution Property
- \( \angle 1 \equiv \angle 3 \)
  - Definition of congruent angles
3. Create a two-column proof of the Congruent Supplement Theorem. Each box of the flow chart proof in Question 2 should appear as a row in the two-column proof.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
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<tr>
<td>2.</td>
<td>2.</td>
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<tr>
<td>3.</td>
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<td>4.</td>
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<td>5.</td>
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<td>7.</td>
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<td>8.</td>
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<td>9.</td>
<td>9.</td>
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</tbody>
</table>

**PROBLEM 5** Proofs of the Congruent Complement Theorem

The **Congruent Complement Theorem** states: “If two angles are complements of the same angle or of congruent angles, then they are congruent.”

1. Draw and label a diagram illustrating this theorem.

2. Use the diagram to write the “Given” and “Prove” statements for the Congruent Complement Theorem.
   
   Given:
   
   Given:
   
   Given:
   
   Prove:
3. Create a flow chart proof of the Congruent Complement Theorem.

4. Create a two-column proof of the Congruent Complement Theorem. Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. The Vertical Angle Theorem states: “Vertical angles are congruent.”

2. Use the diagram to write the “Prove” statements for the Vertical Angle Theorem. The “Given” statements are provided.
   Given: ∠1 and ∠2 are a linear pair
   Given: ∠2 and ∠3 are a linear pair
   Given: ∠3 and ∠4 are a linear pair
   Given: ∠4 and ∠1 are a linear pair
   Prove:
   Prove:
   Prove:

3. Create a flow chart proof of the first “Prove” statement of the Vertical Angle Theorem.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given:</td>
<td></td>
</tr>
<tr>
<td>Given:</td>
<td></td>
</tr>
<tr>
<td>Prove:</td>
<td></td>
</tr>
</tbody>
</table>
Given: \( \angle DEG \equiv \angle HEF \)
Prove: \( \angle DEH \equiv \angle GEF \)

1. Prove the conditional statement using either a paragraph proof or a two-column proof.
1. Which form of proof is easiest to understand? Hardest to understand? Explain.

2. Which form of proof is easiest to write? Hardest to write? Explain.

3. Which form of proof has the fewest steps? The most steps? Explain.


Once a theorem has been proven, it can be used as a reason in another proof. Using theorems that have already been proven allows you to write shorter proofs.

In this chapter, you proved the following theorems:

• The Right Angle Congruence Theorem: All right angles are congruent.
• The Congruent Supplement Theorem: Supplements of congruent angles, or of the same angle are congruent.
• The Congruent Complement Theorem: Complements of congruent angles, or of the same angle are congruent.
• The Vertical Angle Theorem: Vertical angles are congruent.

A list of theorems that you prove throughout this course will be an excellent resource as you continue to make new conjectures and expand our system of geometry.

Be prepared to share solutions and methods.
Introductory Problem for Chapter 3

Fencing for a Garden

A neighbor fenced in her backyard and offers you her leftover fencing. You want to use the fencing to surround a vegetable garden in your yard. The diagram shown is a layout of your property, including measurements and all buildings. Currently, your property has no fencing.

Describe the shape, location, and dimensions of your garden that maximize the area, given the following lengths of fencing.

1. 20 feet of fencing
2. 30 feet of fencing
3. 50 feet of fencing

Be prepared to share your methods and solutions.
5.6 Indirect Measurement

Application of Similar Triangles

OBJECTIVES
In this lesson you will:
• Identify similar triangles to calculate indirect measurements.
• Use proportions to solve for unknown measurements.

KEY TERM
• indirect measurement

PROBLEM 1 How Tall is That Flagpole?

At times, situations occur when measuring something directly is impossible, or physically undesirable. When these situations arise, indirect measurement, the technique that uses proportions to calculate measurement, can be implemented. Your knowledge of similar triangles can be very helpful in these situations.

Use the following steps to measure the height of the school flagpole or any other tall object outside. You will need a partner, a tape measure, a marker, and a flat mirror.

Step 1: Use a marker to create a dot near the center of the mirror.

Step 2: Face the object you would like to measure and place the mirror between yourself and the object. You, the object, and the mirror should be collinear.

Step 3: Focus your eyes on the dot on the mirror and walk backward until you can see the top of the object on the dot, as shown.

Step 4: Ask your partner to sketch a picture of you, the mirror, and the object.
Step 5: Review the sketch with your partner. Decide where to place right angles, and where to locate the sides of the two triangles.

Step 6: Determine which segments in your sketch can easily be measured using the tape measure. Describe their locations and record the measurements on your sketch.

1. How can similar triangles be used to calculate the height of the object?

2. Use your sketch to write a proportion to calculate the height of the object and solve the proportion.

3. Compare your answer with others measuring the same object. How do the answers compare?

4. What are some possible sources of error regard to that could result in using this method?

5. Switch places with your partner and identify a second object to measure. Repeat this method of indirect measurement to solve for the height of the new object.
PROBLEM 2  How Tall is That Oak Tree?

1. You go to the park and use the mirror method to gather enough information to calculate the height of one of the trees. The figure shows your measurements. Calculate the height of the tree.

   ![Diagram of a tree and a person with a mirror]

   5.5 feet
   4 feet
   16 feet

2. Stacey wants to try the mirror method to measure the height of one of her trees. She calculates that the distance between her and the mirror is 3 feet and the distance between the mirror and the tree is 18 feet. Stacey’s eye height is 60 inches. Draw a diagram of this situation. Then calculate the height of this tree.

   Take Note
   Remember, whenever you are solving a problem that involves measurements like length (or weight), you may have to rewrite units so they are the same. For instance, if a problem involves weight, all of the weights should be measured in the same unit of measure.
3. Stacey notices that another tree casts a shadow and suggests that you could also use shadows to calculate the height of the tree. She lines herself up with the tree’s shadow so that the tip of her shadow and the tip of the tree’s shadow meet. She then asks you to measure the distance from the tip of the shadows to her, and then measure the distance from her to the tree. Finally, you draw a diagram of this situation as shown below. Calculate the height of the tree. Explain your reasoning.
PROBLEM 3  How Wide is That Creek?

It is not reasonable for you to directly measure the width of a creek, but you can use indirect measurement to measure the width. You stand on one side of the creek and your friend stands directly across the creek from you on the other side as shown in the figure.

1. Your friend is standing 5 feet from the creek and you are standing 5 feet from the creek. Mark these measurements on the diagram shown.

2. You and your friend walk away from each other in opposite parallel directions. Your friend walks 50 feet and you walk 12 feet. Mark these measurements on the diagram shown. Draw a line segment that connects your starting point and ending point and draw a line segment that connects your friend’s starting point and ending point.

3. Draw a line segment that connects you and your friend’s starting points and draw a line segment that connects you and your friend’s ending points. Label any angle measures and any angle relationships that you know on the diagram. Explain how you know these angle measures.

4. How do you know that the triangles formed by the lines are similar?
5. Calculate the distance from your friend’s starting point to your side of the creek. Round your answer to the nearest tenth, if necessary.

6. What is the width of the creek? Explain your reasoning.

7. There is also a ravine (a deep hollow in the earth) on another edge of the park. You and your friend take measurements like those in Problem 3 to indirectly calculate the width of the ravine. The figure shows your measurements. Calculate the width of the ravine.
8. There is a large pond in the park. A diagram of the pond is shown below. You want to calculate the distance across the widest part of the pond, labeled as $DE$. To indirectly calculate this distance, you first place a stake at point $A$. You chose point $A$ so that you can see the edge of the pond on both sides at points $D$ and $E$, where you also place stakes. Then you tie string from point $A$ to point $D$ and from point $A$ to point $E$. At a narrow portion of the pond, you place stakes at points $B$ and $C$ along the string so that $BC$ is parallel to $DE$. The measurements you make are shown on the diagram. Calculate the distance across the widest part of the pond.

Be prepared to share your solutions and methods.
OBJECTIVES
In this lesson you will:
- Write the inverse of a conditional statement.
- Differentiate between direct and indirect proof.
- Use indirect proof.

KEY TERMS
- inverse
- contrapositive
- direct proof
- indirect proof or proof by contradiction
- Hinge Theorem
- Hinge Converse Theorem

PROBLEM 1 The Inverse and Contrapositive

Every conditional statement written in the form “If \( p \), then \( q \)” has three additional conditional statements associated with it: converse, contrapositive, and inverse. Recall from previous lessons, to state the converse, reverse the hypothesis, \( p \), and the conclusion, \( q \). To state the inverse, negate both parts. To state the contrapositive, negate each part and reverse them.

<table>
<thead>
<tr>
<th>Conditional Statement</th>
<th>If ( p ), then ( q ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converse</td>
<td>If ( q ), then ( p ).</td>
</tr>
<tr>
<td>Inverse</td>
<td>If not ( p ), then not ( q ).</td>
</tr>
<tr>
<td>Contrapositive</td>
<td>If not ( q ), then not ( p ).</td>
</tr>
</tbody>
</table>

For each conditional statement written in propositional form, identify the hypothesis \( p \) and conclusion \( q \). Identify the negation of the hypothesis and conclusion, and then write the inverse and contrapositive of the conditional statement.

1. If a quadrilateral is a square, then the quadrilateral is a rectangle.
   a. Hypothesis \( p \):
   b. Conclusion \( q \):
   c. Is the conditional statement true? Explain.
   d. Not \( p \):
e. Not $q$:

f. Inverse:

g. Is the inverse true? Explain.

h. Contrapositive:

i. Is the contrapositive true? Explain.

2. If an integer is even, then the integer is divisible by two.

a. Hypothesis $p$:

b. Conclusion $q$:

c. Is the conditional statement true? Explain.

d. Not $p$:

e. Not $q$:

f. Inverse:

g. Is the inverse true? Explain.

h. Contrapositive:

i. Is the contrapositive true? Explain.
3. If a polygon has six sides, then the polygon is a pentagon.
   a. Hypothesis $p$:
   b. Conclusion $q$:
   c. Is the conditional statement true? Explain.
   d. Not $p$:
   e. Not $q$:
   f. Inverse:
   g. Is the inverse true? Explain.
   h. Contrapositive:
   i. Is the contrapositive true? Explain.

4. If two lines intersect, then the lines are perpendicular.
   a. Hypothesis $p$:
   b. Conclusion $q$:
   c. Is the conditional statement true? Explain.
   d. Not $p$:
e. Not $q$:

f. Inverse:

g. Is the inverse true? Explain.

h. Contrapositive:

i. Is the contrapositive true? Explain.

5. What do you notice about the truth value of a conditional statement and the truth value of its inverse?

6. What do you notice about the truth value of a conditional statement and the truth value of its contrapositive?
All of the true statements up to this point were proven directly. A **direct proof** begins with the given information and works to the desired conclusion directly through the use of givens, definitions, properties, postulates, and theorems.

An indirect proof is different and may be shorter than a direct proof. An **indirect proof**, or **proof by contradiction**, uses the contrapositive. If you prove the contrapositive true, then the statement is true. Begin by assuming the conclusion is false and use this assumption to show one of the given statements is false, thereby creating a contradiction.

In an indirect proof:
- Write the givens.
- Write the negation of the conclusion.
- Use the assumption, in conjunction with definitions, properties, postulates, and theorems, to prove a given statement is false, thus creating a contradiction.

Hence, your assumption leads to a contradiction; therefore, the assumption must be false. This proves the contrapositive.

Let’s look at an example of an indirect proof.

**Given:** In \( \triangle CHT \), \( \overline{CA} \) does not bisect \( \overline{HT} \)

**Prove:** \( \triangle CHA \neq \triangle CTA \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle CHA \equiv \triangle CTA )</td>
<td>1. Assumption</td>
</tr>
<tr>
<td>2. ( \overline{CA} ) does not bisect ( \overline{HT} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \overline{HA} \equiv \overline{TA} )</td>
<td>3. CPCTC</td>
</tr>
<tr>
<td>4. ( \overline{CA} ) bisects ( \overline{HT} )</td>
<td>4. Definition of bisect</td>
</tr>
<tr>
<td>5. ( \triangle CHA \equiv \triangle CTA ) is false</td>
<td>5. Step 4 contradicts step 2; the assumption is false</td>
</tr>
<tr>
<td>6. ( \triangle CHA \neq \triangle CTA ) is true</td>
<td>6. Proof by contradiction</td>
</tr>
</tbody>
</table>

In step 5, the “assumption” is stated as “false.” The reason for making this statement is “contradiction.”
Now try one yourself!

1. Given: $BR$ bisects $\angle ABN$
   
   $\angle BRA \cong \angle BRN$
   
   Prove: $\overline{AB} \ncong \overline{NB}$

<table>
<thead>
<tr>
<th>Statements</th>
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<tbody>
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</table>

2. When writing an indirect proof, it is often easier to write it as a paragraph proof. Write the proof in Question 1 as a paragraph proof.

3. Use a paragraph proof model to write an indirect proof proving a triangle cannot have more than one right angle.
The Hinge Theorem states: "If two sides of one triangle are congruent to two sides of another triangle and the included angle of the first pair is larger than the included angle of the second pair, then the third side of the first triangle is longer than the third side of the second triangle."

In the two triangles shown, notice that $RS = DE$, $ST = EF$, and $\angle S > \angle E$. The Hinge Theorem guarantees that $RT > DE$.

1. Use an indirect proof to prove the Hinge Theorem.
   Begin by restating the Hinge Theorem using $\triangle ABC$ and $\triangle DEF$.
   If sides $AB = DE$ and $AC = DF$, and the included angle at $A$ is larger than the included angle at $D$, then $BC > EF$.

   Given: $AB = DE$
   $AC = DF$
   $m\angle A > m\angle D$
   Prove: $BC > EF$
This theorem must be proven for two cases:
Case 1: $BC = EF$
Case 2: $BC < EF$

a. Write the indirect proof for Case 1.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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</tbody>
</table>

b. Write the indirect proof for Case 2.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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</thead>
<tbody>
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<td></td>
<td></td>
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</tbody>
</table>
The **Hinge Converse Theorem** states: “If two sides of one triangle are congruent to two sides of another triangle and the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first pair of sides is larger than the included angle of the second pair of sides.”

In the two triangles shown, notice that $RT = DF$, $RS = DE$, and $ST > EF$. The Hinge Converse Theorem guarantees that $\angle R > \angle D$.

2. Create an indirect proof to prove the Hinge Converse Theorem.

Given: $AB = DE$
$AC = DF$
$BC > EF$

Prove: $\angle A > \angle D$

This theorem must be proven for two cases:
Case 1: $\angle A = \angle D$
Case 2: $\angle A < \angle D$

a. Create an indirect proof for Case 1.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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</table>
3. Use the Hinge Theorem and its converse to answer each.

a. Matthew and Jeremy’s families are going camping for the weekend. Before heading out of town, they decide to meet at Al’s Diner for breakfast. During breakfast, the boys try to decide which family will be further away from the diner “as the crow flies.” “As the crow flies” is an expression based on the fact that crows, generally fly straight to the nearest food supply.

Matthew’s family is driving 35 miles due north and taking an exit to travel an additional 15 miles northeast. Jeremy’s family is driving 35 miles due south and taking an exit to travel an additional 15 miles southwest. Use the diagram shown to determine which family is further from the diner. Explain your reasoning.
b. Which of the following is a possible length for \( AH \): 20 cm, 21 cm, or 24 cm? Explain your choice.

![Diagrams of triangles with angles and side lengths]

\( \angle EPW = 55^\circ \)
\( \angle ARH = 61^\circ \)
\( AH = 21 \text{ cm} \)

---

c. Which of the following is a possible angle measure for \( \angle ARH \): 54º, 55º or 56º? Explain your choice.

![Diagrams of triangles with angles and side lengths]

\( \angle EPW = 55^\circ \)
\( \angle ARH = 61^\circ \)
\( AH = 34 \text{ mm} \), \( AH = 36 \text{ mm} \)

---

Be prepared to share your solutions and methods.
In geometry, it is necessary to know the sum of the interior angles of various polygons to determine other information. Is there a quick method for calculating the sum of the measures of different polygons? Let’s find out!

A teacher asked her class to determine the sum of the measures of the interior angles of a quadrilateral and noticed that two of her students, Carson and Juno, were already engaged in a heated disagreement.
Carson had drawn a quadrilateral and added one diagonal. He concluded that the sum of the measures of the interior angles of a quadrilateral must be equal to $360^\circ$.

1. Describe Carson’s reasoning.

Juno had drawn a quadrilateral and added two diagonals. She concluded that the sum of the measures of the interior angles of a quadrilateral must be equal to $720^\circ$.

2. Describe Juno’s reasoning.

3. Who is correct? Explain.
As always, you must start with what you know to be true. The Triangle Sum Theorem states that the sum of the three interior angles of any triangle is equal to 180°. You can use this information to calculate the sum of the interior angles of other polygons.

**Quadrilaterals**

**Step 1:** Draw a quadrilateral.

**Step 2:** Use only one vertex of the quadrilateral, draw all possible diagonals. Remember, a diagonal is a line segment connecting non-adjacent vertices.

**Step 3:** The diagonal(s) of the quadrilateral divided it into how many triangles?

**Step 4:** If the sum of the measures of the interior angles of each triangle is 180°, what is the sum of the measures of all of the interior angles of the triangles formed by the diagonal(s)?
Pentagons

Step 1: Draw a pentagon.

Step 2: Use only one vertex of the pentagon, draw all possible diagonals.

Step 3: The diagonal(s) of the pentagon divided it into how many triangles?

Step 4: If the sum of the measures of the interior angles of each triangle is 180°, what is the sum of the measures of all of the interior angles of the triangles formed by the diagonal(s)?

Hexagons

Step 1: Draw a hexagon.
Step 2: Use only one vertex of the hexagon, draw all possible diagonals.

Step 3: The diagonal(s) of the hexagon divided it into how many triangles?

Step 4: If the sum of the measures of the interior angles of each triangle is $180^\circ$, what is the sum of the measures of all of the interior angles of the triangles formed by the diagonal(s)?

1. Complete the table.

<table>
<thead>
<tr>
<th>Number of sides of the polygon</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of diagonals drawn</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of triangles formed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of the measures of the interior angles</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

2. What is the relationship between the number of possible diagonals drawn from one vertex of the polygon and the number of triangles formed by those diagonals?

3. Compare the number of sides of the polygon to the number of possible diagonals drawn from one vertex. What do you notice?

4. Compare the number of sides of the polygon to the number of triangles formed by extending all possible diagonals from one vertex. What do you notice?
5. What pattern do you notice about the sum of the measures of the interior angles of a polygon as the number of sides of each polygon increases by 1?

6. Use the chart. Can you predict the number of possible diagonals drawn from one vertex and the number of triangles formed for a seven-sided polygon?

7. Use the chart. Can you predict the sum of the measures of the interior angles of a seven-sided polygon?

8. Continue the pattern to complete the chart.

<table>
<thead>
<tr>
<th>Number of sides of the polygon</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of diagonals drawn</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of triangles formed</td>
<td></td>
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</tr>
<tr>
<td>Sum of the measures of the interior angles</td>
<td></td>
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</tbody>
</table>

9. When you calculated the number of triangles formed in the 16-sided polygon, did you need to know how many triangles were formed in a 15-sided polygon first? Explain.

10. If a polygon has 100 sides, how many triangles are formed by extending all possible diagonals from one vertex? Explain.

11. What is the sum of the measures of the interior angles of a 100-sided polygon? Explain.

12. If a polygon has $n$ sides, how many triangles are formed by extending all diagonals from one vertex? Explain.

13. What is the sum of the measures of the interior angles of an $n$-sided polygon? Explain.
14. Use the formula to calculate the sum of the measures of the interior angles of a polygon with 32 sides.

15. If the sum of the measures of the interior angles of a polygon is $9540^\circ$, how many sides does the polygon have? Explain your reasoning.

**PROBLEM 3**

**Sum of the Measures of the Interior Angles of a Regular Polygon**

1. Use the formula developed in Problem 1 to calculate the sum of the measures of the interior angles of a decagon.

2. Calculate the measure of each interior angle of a decagon if each interior angle is congruent. How did you get this answer?

3. Complete the chart.

<table>
<thead>
<tr>
<th>Number of sides of regular polygon</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of measures of interior angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure of each interior angle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. If a regular polygon has $n$ sides, write a formula to calculate the measure of each interior angle.

5. Use the formula to determine the measure of each interior angle of a regular 100-sided polygon.
6. If the measure of each interior angle of a regular polygon is equal to 150º, determine the number of sides. How did you get this answer?

7. Apply what you have learned on the star shown. *PENTA* is a regular pentagon. Solve for *x*.

Be prepared to share your methods and solutions.
Calculating Perimeters and Areas of Rectangles

To calculate the perimeter of a rectangle, multiply the length by 2 and the width by 2, and then calculate the sum of the products. To calculate the area of a rectangle, multiply the length by the width.

Example:

\[ P = 2\ell + 2w = 2(6) + 2(4) = 20 \text{ feet} \]
\[ A = \ell w = 6(4) = 24 \text{ square feet} \]

The perimeter of the rectangle is 20 feet, and the area of the rectangle is 24 square feet.

KEY TERMS
- rectangle (3.1)
- square (3.1)
- parallelogram (3.2)
- altitude of a parallelogram (3.2)
- height of a parallelogram (3.2)
- altitude of a triangle (3.2)
- height of a triangle (3.2)
- legs (3.3)
- altitude of a trapezoid (3.3)
- height of a trapezoid (3.3)
- isosceles trapezoid (3.3)
- congruent polygons (3.4)
- apothem (3.4)
- circle (3.5)
- diameter (3.5)
- radius (3.5)
- circumference (3.5)
- irrational number (3.5)
- concentric circles (3.5)
- annulus (3.5)
- composite figure (3.6)

CONSTRUCTIONS
- square given perimeter (3.1)
- rectangle given perimeter (3.1)
- isosceles triangle given perimeter (3.2)
- trapezoid (3.3)
### 3.1 Calculating Perimeters and Areas of Squares

To calculate the perimeter of a square, multiply the length of a side by 4. To calculate the area of a square, multiply the length of a side by itself.

**Example:**

```
12 m

P = 4s = 4(12) = 48 meters
A = s \cdot s = s^2 = 12^2 = 144 square meters
```

The perimeter of the square is 48 meters, and the area of the square is 144 square meters.

### 3.2 Calculating Areas of Parallelograms

To calculate the area of a parallelogram, multiply the base by the height.

**Example:**

```
10.5 m

A = bh
   = 22(10.5)
   = 231 square meters
```

The area of the parallelogram is 231 square meters.
### 3.2 Calculating Areas of Triangles

To calculate the area of a triangle, multiply one half of the base by the height.

**Example:**

\[A = \frac{1}{2}bh\]

\[= \frac{1}{2}(11)(7)\]

\[= 38.5\text{ square centimeters}\]

The area of the triangle is 38.5 square centimeters.

### 3.3 Calculating Areas of Trapezoids

To calculate the area of a trapezoid, multiply one half of the sum of the bases by the height.

**Example:**

\[A = \frac{1}{2}(b_1 + b_2)h\]

\[= \frac{1}{2}(35 + 65)(30)\]

\[= 1500\text{ square yards}\]

The area of the trapezoid is 1500 square yards.
3.4 Calculating Areas of Regular Polygons

To calculate the area of a regular polygon, multiply one half of the length of one side of the polygon times the length of the apothem times the number of sides.

Example:

\[ A = \frac{1}{2} (a) n \]
\[ = \frac{1}{2} (7)(4.8)(5) \]
\[ = 84 \text{ square inches} \]

The area of the regular pentagon is 84 square inches.

3.5 Calculating Circumferences and Areas of Circles

To calculate the circumference of a circle, multiply the diameter by π. You can also multiply the radius by 2π. To calculate the area of a circle, multiply the square of the radius by π. You can use 3.14 to approximate π.

Example:

\[ C = \pi d = 2\pi r \]
\[ \approx 2(3.14)(3.25) \]
\[ \approx 20.41 \text{ feet} \]

\[ A = \pi r^2 \]
\[ \approx (3.14)(3.25^2) \]
\[ \approx 33.17 \text{ square feet} \]

The circumference of the circle is approximately 20.41 feet and the area of the circle is approximately 33.17 square feet.
3.6 Calculating Areas of Composite Figures

To calculate the area of a composite figure, calculate the area of each common figure that makes up the composite figure and then calculate the sum of the areas.

Example

You have a flower bed in a corner of your backyard. A diagram of the flower bed is shown. You want to cover the flower bed with mulch. What is the area that you will be covering with mulch?

Area of square = (4)(4) = 16 square feet
Area of rectangle = (7)(4) = 28 square feet
Area of quarter of circle = \( \frac{1}{4} \pi (4)^2 = \frac{1}{4} (3.14)(16) = 12.56 \) square feet
Area of composite figure = 16 + 28 + 12.56 = 56.56 square feet

You will be covering an area of about 56.56 square feet with mulch.
GEOMETRY

Teacher’s Implementation Guide
OBJECTIVES
In this lesson you will:
• Write a formula for the sum of the interior angles of any polygon.
• Calculate the sum of the interior angles of any polygon, given the number of sides.
• Calculate the number of sides of a polygon, given the sum of the interior angles.
• Write a formula for the measure of each interior angle of any regular polygon.
• Calculate the measure of an interior angle of a regular polygon, given the number of sides.
• Calculate the number of sides of a regular polygon, given the sum of the interior angles.

KEY TERM
• interior angle of a polygon

NCTM CONTENT STANDARDS
Algebra Standard
• Use symbolic algebra to represent and explain mathematical relationships.

Geometry Standards
• Analyze properties and determine attributes of two- and three-dimensional objects.
• Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

ESSENTIAL IDEAS
• The sum of the measures of the interior angles of a polygon is $180(n - 2)$, where $n$ is the number of sides.
• The measure of each interior angle of a regular polygon is $\frac{180(n - 2)}{n}$, where $n$ is the number of sides.

ESSENTIAL QUESTIONS
1. How do you calculate the sum of the measures of the interior angles of a polygon?
2. How do you calculate the measure of an interior angle of a regular polygon?
Show The Way

Warm Up

Solve for \( n \).

1. \( 180(5 - 1) = n \)
   \[ 180(4) = n \]
   \[ n = 720 \]

2. \( 180(n - 1) = 900 \)
   \[ 180n - 180 = 900 \]
   \[ 180n = 1080 \]
   \[ n = 6 \]

3. \( \frac{180(3 - 1)}{4} = n \)
   \[ \frac{180(2)}{4} = n \]
   \[ 360 = n \]
   \[ n = 9 \]

4. \( \frac{180(n - 1)}{n} = 120 \)
   \[ 180n - 180 = 120n \]
   \[ 180n - 180 = 120n \]
   \[ 60n = 180 \]
   \[ n = 3 \]

Motivator

John announced to the class that he could calculate the sum of the measures of the starred angles in this diagram without knowing the measure of any specific angle. How is this possible? Using theorems or postulates, explain what John is thinking.

Using the Linear Pair Postulate, John would first determine that the sum of the measures of all 12 angles formed by the three intersecting lines is \( 180(6) = 1080^\circ \). John would then use the Triangle Sum Theorem to determine that the sum of the interior angles of the triangle is \( 180^\circ \). By subtracting the sum of the interior angles of the triangle from the sum of the 12 angles, John would then determine the difference, which is the sum of the starred angles.
Problem 1

This activity provides students with the opportunity to develop the formula used to determine the sum of the measures of the interior angles of a polygon: \(180(n - 2)\), where \(n\) is the number of sides. Students will use prior knowledge, specifically the Triangle Sum Theorem, to gather data related to number of sides of the polygon, number of diagonals drawn from one vertex, number of triangles formed, and sum of the measures of the interior angles of the total number of triangles formed by the diagonals. Viewing this data in a chart format allows students to identify a pattern and eventually generalize. This generalization enables the students to create a formula for polygons with \(n\) sides.

Grouping

Allow the students to work in pairs or groups on Questions 1 through 3. Bring the class together to compare its work and check for understanding.
Carson had drawn a quadrilateral and added one diagonal. He concluded that the sum of the measures of the interior angles of a quadrilateral must be equal to 360º.

1. Describe Carson’s reasoning.
Carson concluded the sum of the measures of the interior angles of a quadrilateral is equal to 360º because the diagonal formed two distinct triangles within the quadrilateral and the sum of the measures of the interior angles of each triangle is 180º, so $2(180º) = 360º$.

Juno had drawn a quadrilateral and added two diagonals. She concluded that the sum of the measures of the interior angles of a quadrilateral must be equal to 720º.

2. Describe Juno’s reasoning.
Juno concluded the sum of the measures of the interior angles of a quadrilateral is equal to 720º because the intersecting diagonals formed four distinct triangles within the quadrilateral and the sum of the measures of the interior angles of each triangle is 180º, so $4(180º) = 720º$.

3. Who is correct? Explain.
Carson is correct. When drawing the two intersecting diagonals, Juno created extra angles that are not considered interior angles of the original quadrilateral, so her answer has an extra 360º because the additional angles form a circle.

Note
Students do not need a calculator, but use of a calculator is not discouraged. The focus of this lesson is not recall of multiplication facts. If we can minimize arithmetic errors, it will be easier to accomplish our objectives.
**Problem 2**

**Grouping**
Allowing students the time to gather data and develop their own formula will empower them to reconstruct their knowledge when needed, rather than memorize the formula and not understand why you subtract 2 from the number of sides or err in trying to recall what number was subtracted.

**Note**
The questions in this activity require students to work arithmetically both forward and backward. Students will be given the number of sides of the polygon and asked to solve for the sum of the measures of the interior angles of the polygon, as well as solve for the number of sides in the polygon given the sum of the measures of the interior angles.
Chapter 8 | Quadrilaterals

Pentagons

Step 1: Draw a pentagon.

Step 2: Use only one vertex of the pentagon, draw all possible diagonals.

Step 3: The diagonal(s) of the pentagon divided it into how many triangles?
   The diagonals divided the pentagon into three triangles.

Step 4: If the sum of the measures of the interior angles of each triangle is 180°, what is the sum of the measures of all of the interior angles of the triangles formed by the diagonal(s)?
   The sum of the measures of the interior angles of the three triangles is 540°.

Hexagons

Step 1: Draw a hexagon.
Step 2: Use only one vertex of the hexagon, draw all possible diagonals.

Step 3: The diagonal(s) of the hexagon divided it into how many triangles?
The diagonals divided the hexagon into four triangles.

Step 4: If the sum of the measures of the interior angles of each triangle is 180º, what is the sum of the measures of all of the interior angles of the triangles formed by the diagonal(s)?
The sum of the measures of the interior angles of the four triangles is 720º.

1. Complete the table.

<table>
<thead>
<tr>
<th>Number of sides of the polygon:</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of diagonals drawn</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Number of triangles formed</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Sum of the measures of the interior angles</td>
<td>180º</td>
<td>360º</td>
<td>540º</td>
<td>720º</td>
</tr>
</tbody>
</table>

2. What is the relationship between the number of possible diagonals drawn from one vertex of the polygon and the number of triangles formed by those diagonals?
   One diagonal forms two triangles, two diagonals form three triangles, and so on. There is always one more triangle than the number of diagonals.

3. Compare the number of sides of the polygon to the number of possible diagonals drawn from one vertex. What do you notice?
   The number of diagonals drawn is always three less than the number of sides of the polygon.

4. Compare the number of sides of the polygon to the number of triangles formed by extending all possible diagonals from one vertex. What do you notice?
   The number of triangles formed is always two less than the number of sides of the polygon.
5. What pattern do you notice about the sum of the measures of the interior angles of a polygon as the number of sides of each polygon increases by 1?
   As the number of sides of each polygon increases by 1, the sum of the measures of the interior angles of the polygon increases by 180°.

6. Use the chart. Can you predict the number of possible diagonals drawn from one vertex and the number of triangles formed for a seven-sided polygon?
   A seven-sided polygon will have four diagonals extended from one vertex and it will be divided into five triangles.

7. Use the chart. Can you predict the sum of the measures of the interior angles of a seven-sided polygon?
   The sum of the measures of the interior angles of a seven-sided polygon will be 900°.

8. Continue the pattern to complete the chart.

<table>
<thead>
<tr>
<th>Number of sides of the polygon</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of diagonals drawn</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Number of triangles formed</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>Sum of the measures of the interior angles</td>
<td>900°</td>
<td>1080°</td>
<td>1260°</td>
<td>2520°</td>
</tr>
</tbody>
</table>

9. When you calculated the number of triangles formed in the 16-sided polygon, did you need to know how many triangles were formed in a 15-sided polygon first? Explain.
   No, I knew there were two fewer triangles formed than the number of sides, so I just subtracted 2 from 16 to get the number of triangles formed.

10. If a polygon has 100 sides, how many triangles are formed by extending all possible diagonals from one vertex? Explain.
    A 100-sided polygon will be divided into 98 triangles. There are always two fewer triangles than the number of sides.

11. What is the sum of the measures of the interior angles of a 100-sided polygon?
    Explain.
    The sum of the measure of the interior angles of a 100-sided polygon is 17640°. You multiply the number of triangles by 180°.

12. If a polygon has n sides, how many triangles are formed by extending all diagonals from one vertex? Explain.
    An n-sided polygon will be divided into (n – 2) triangles. There are always two fewer triangles than the number of sides.

13. What is the sum of the measures of the interior angles of an n-sided polygon?
    Explain.
    The sum of the measure of the interior angles of an n-sided polygon is 180° (n – 2). You multiply the number of triangles by 180°.
Problem 3

Students use their new knowledge from Problem 2 to derive a formula that determines the measure of each interior angle of a regular polygon: \(\frac{180(n - 2)}{n}\), where \(n\) is the number of sides. A chart is used to organize their data and pattern recognition guides the students to a generalization.

Students will be given the number of sides of the regular polygon and asked to solve for the measure of each interior angle, as well as solve for the number of sides in the regular polygon given the measure of each interior angle.

Grouping

Allow the students to work in pairs or groups on Questions 1 through 7. Bring the class together to compare its work and check for understanding.

Note

In Question 4 students will derive the formula to calculate the measure of the each interior angle of a regular polygon.

14. Use the formula to calculate the sum of the measures of the interior angles of a polygon with 32 sides.
   The sum of the measures of the interior angles of a polygon with 32 sides is \(180(30) = 5400\)º.

15. If the sum of the measures of the interior angles of a polygon is 9540º, how many sides does the polygon have? Explain your reasoning.
   The polygon has 55 sides. I divided 9540 by 180 and added 2.

PROBLEM 3  Sum of the Measures of the Interior Angles of a Regular Polygon

1. Use the formula developed in Problem 1 to calculate the sum of the measures of the interior angles of a decagon.
   \(180(n - 2) = 180(10 - 2) = 1440\)º

2. Calculate the measure of each interior angle of a decagon if each interior angle is congruent. How did you get this answer?
   The measure of each interior angle of the decagon is equal to 144º. I divided the sum of the measures of the interior angles by the number of sides (10).

3. Complete the chart.

<table>
<thead>
<tr>
<th>Number of sides of regular polygon</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of measures of interior angles</td>
<td>180º</td>
<td>360º</td>
<td>540º</td>
<td>720º</td>
<td>900º</td>
<td>1080º</td>
</tr>
<tr>
<td>Measure of each interior angle</td>
<td>60º</td>
<td>90º</td>
<td>108º</td>
<td>120º</td>
<td>128.57º</td>
<td>135º</td>
</tr>
</tbody>
</table>

4. If a regular polygon has \(n\) sides, write a formula to calculate the measure of each interior angle.
   \(\frac{180(n - 2)}{n}\)

5. Use the formula to determine the measure of each interior angle of a regular 100-sided polygon.
   \(\frac{180(100 - 2)}{100} = 176.4º\)
Note

Question 7 requires the students to apply the Triangle Sum Theorem and the Linear Pair Postulate in addition to using the new formula to determine the measure of an angle embedded in a five-point star.

Essential Ideas

- The sum of the measures of the interior angles of a polygon is \(180(n - 2)\), where \(n\) is the number of sides.
- The measure of each interior angle of a regular polygon is \(\frac{180(n - 2)}{n}\), where \(n\) is the number of sides.

6. If the measure of each interior angle of a regular polygon is equal to 150º, determine the number of sides. How did you get this answer?

The regular polygon has 12 sides.

I worked the formula backwards.

\[
180 \left( \frac{n - 2}{n} \right) = 150 \\
180n - 360 = 150n \\
30n = 360 \\
n = 12
\]

7. Apply what you have learned on the star shown. PENTA is a regular pentagon. Solve for \(x\).

By adding two diagonals in the pentagon, you can determine that the sum of the measures of the interior angles is equal to 540º. Each interior angle must be equal because the pentagon is regular, so each interior angle is 108º. Each interior angle of the pentagon is an exterior angle of a triangle, forming a linear pair with one interior angle of a triangle. Therefore each interior angle in the linear pair must be 72º. If two angles in each triangle are 72º, that leaves 36º for the third angle in the triangle, so \(x = 36º\).

Be prepared to share your methods and solutions.
Follow Up

Assignment
Use the Assignment for Lesson 8.4 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 8.4 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 8.

Check Students’ Understanding
Use the six-pointed star and the regular hexagon $HEXAGO$ to solve for $n$.

Each interior angle in the regular hexagon is $120^\circ$. Using the Linear Pair Postulate, it can be determined that two of the interior angles in the bottom triangle each have a measure $60^\circ$. Using the Triangle Sum Theorem, the third angle must be $60^\circ$, so $n = 60^\circ$. 

GEOMETRY

Teacher’s Resources and Assessments

Carnegie Learning
Decomposing Polygons
Sum of the Measures of the Interior Angles of a Polygon

Determine the measure of an interior angle of each regular polygon.

1. regular nonagon

\[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(9 - 2)}{9} = \frac{1260^\circ}{9} = 140^\circ
\]

2. regular decagon

\[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(10 - 2)}{10} = \frac{1440^\circ}{10} = 144^\circ
\]

3. regular 15-gon

\[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(15 - 2)}{15} = \frac{2340^\circ}{15} = 156^\circ
\]

4. regular 47-gon

\[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(47 - 2)}{47} = \frac{8100^\circ}{47} \approx 172.34^\circ
\]

Determine the measure of the missing angle in each figure.

5.

\[
720^\circ - (166^\circ + 108^\circ + 121^\circ + 135^\circ + 90^\circ) = 100^\circ
\]
The measure of the missing angle is 100°.

6.

\[
1080^\circ - (128^\circ + 99^\circ + 161^\circ + 113^\circ + 142^\circ + 146^\circ + 135^\circ) = 156^\circ
\]
The measure of the missing angle is 156°.
7. Use the figure to answer each question.

a. What is the sum of the measures of the interior angles of the polygon?

\[(n - 2)(180°) = (5 - 2)(180°) = 3(180°) = 540°\]

The sum of the measures of the interior angles of the polygon is 540°.

b. What is the value of \(x\)?

\[2x + 7x + 4x + 4x + 5x - 12 + 2 = 540\]
\[22x - 10 = 540\]
\[22x = 550\]
\[x = 25\]

c. What is the measure of \(\angle PTS\)?

\[m\angle PTS = (5x)^\circ = (5 \cdot 25)^\circ = 125°\]

The measure of \(\angle PTS\) is 125°.

d. What is the measure of \(\angle RQP\)?

\[m\angle RQP = (7x - 12)^\circ = (7(25) - 12)^\circ = 163°\]

The measure of \(\angle RQP\) is 163°.
8. Suppose that the sum of the interior angles of a regular polygon is 157.5°. What type of polygon is it? Show your work and explain how you got your answer.

The formula for the measure of an interior angle of a regular polygon is \( \frac{180(n - 2)}{n} \), where \( n \) represents the number of sides of the polygon. So, set the formula equal to 157.5° to determine the number of sides.

\[
\frac{180(n - 2)}{n} = 157.5
\]

\[
180n - 360 = 157.5n
\]

\[
22.5n = 360
\]

\[
n = 16
\]

The polygon has 16 sides. So, the polygon is a 16-gon.

9. Suppose that the measure in degrees of each angle of a regular 12-gon can be represented by the expression \( 2x + 5 \). Calculate the value of \( x \).

\[
\frac{180(n - 2)}{n} = \frac{180(12 - 2)}{12} = \frac{1800^\circ}{12} = 150^\circ
\]

\[
2x + 5 = 150
\]

\[
2x = 145
\]

\[
x = 72.5
\]
Skills Practice

Decomposing Polygons
Sum of the Measures of the Interior Angles of a Polygon

Vocabulary
Define each term in your own words.

1. Triangle Sum Theorem
   The Triangle Sum Theorem states that the sum of the measures of the interior angles of any triangle is 180°.

2. Polygon
   A polygon is a two-dimensional figure that is formed by three or more segments called sides. Each side of a polygon must intersect exactly two other sides, one at each endpoint. No two sides can intersect each other more than once.

3. Interior angle of a polygon
   An interior angle of a polygon is an angle that is formed by two consecutive sides of a polygon.

Problem Sets
Using only one vertex, draw all possible diagonals for each polygon, and then write the total number of triangles formed by the diagonals.

1. 3 triangles
2. 4 triangles
Determine the sum of the measures of the interior angles of each type of polygon.

3. 2 triangles

5. 6 triangles

4. 2 triangles

6. 5 triangles

7. trapezoid
\[(n - 2)(180°) = (4 - 2)(180°)
= 2(180°) = 360°\]

8. hexagon
\[(n - 2)(180°) = (6 - 2)(180°)
= 4(180°) = 720°\]

9. pentagon
\[(n - 2)(180°) = (5 - 2)(180°)
= 3(180°) = 540°\]

10. nonagon
\[(n - 2)(180°) = (9 - 2)(180°)
= 7(180°) = 1260°\]

11. 12-gon
\[(n - 2)(180°) = (12 - 2)(180°)
= 10(180°) = 1800°\]

12. 20-gon
\[(n - 2)(180°) = (20 - 2)(180°)
= 18(180°) = 3240°\]

13. 36-gon
\[(n - 2)(180°) = (36 - 2)(180°)
= 34(180°) = 6120°\]

14. 52-gon
\[(n - 2)(180°) = (52 - 2)(180°)
= 50(180°) = 9000°\]
Use a formula to determine the measure of an interior angle of each regular polygon.

15. \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(4 - 2)}{4} = \frac{360^\circ}{4} = 90^\circ
\]

16. \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(8 - 2)}{8} = \frac{1080^\circ}{8} = 135^\circ
\]

17. \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(6 - 2)}{6} = \frac{720^\circ}{6} = 120^\circ
\]

18. \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(5 - 2)}{5} = \frac{540^\circ}{5} = 108^\circ
\]

19. 17-gon \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(17 - 2)}{17} = \frac{2700^\circ}{17} \approx 158.82^\circ
\]

20. 25-gon \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(25 - 2)}{25} = \frac{4140^\circ}{25} = 165.6^\circ
\]

21. 60-gon \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(60 - 2)}{60} = \frac{10,440^\circ}{60} = 174^\circ
\]

22. 84-gon \[
\frac{180^\circ(n - 2)}{n} = \frac{180^\circ(84 - 2)}{84} = \frac{14,760^\circ}{84} \approx 175.71^\circ
\]
The measure of an interior angle of a regular polygon is given. Determine the number of sides of each polygon.

23. 144°

\[
\frac{180(n - 2)}{n} = 144
\]

\[
180n - 360 = 144n
\]

\[
36n = 360
\]

\[
n = 10
\]

24. 156°

\[
\frac{180(n - 2)}{n} = 156
\]

\[
180n - 360 = 156n
\]

\[
24n = 360
\]

\[
n = 15
\]

25. 168°

\[
\frac{180(n - 2)}{n} = 168
\]

\[
180n - 360 = 168n
\]

\[
12n = 360
\]

\[
n = 30
\]

26. 165°

\[
\frac{180(n - 2)}{n} = 165
\]

\[
180n - 360 = 165n
\]

\[
15n = 360
\]

\[
n = 24
\]

27. 177°

\[
\frac{180(n - 2)}{n} = 177
\]

\[
180n - 360 = 177n
\]

\[
3n = 360
\]

\[
n = 120
\]

28. 172°

\[
\frac{180(n - 2)}{n} = 172
\]

\[
180n - 360 = 172n
\]

\[
8n = 360
\]

\[
n = 45
\]

29. 174.375°

\[
\frac{180(n - 2)}{n} = 174.375
\]

\[
180n - 360 = 174.375n
\]

\[
5.625n = 360
\]

\[
n = 64
\]

30. 177.6°

\[
\frac{180(n - 2)}{n} = 177.6
\]

\[
180n - 360 = 177.6n
\]

\[
2.4n = 360
\]

\[
n = 150
\]
Pre-Test

Name ___________________________________________________ Date _____________________

Match each polygon to the correct number of its sides.

1. hexagon A. 4 sides
2. pentagon B. 10 sides
3. quadrilateral C. 9 sides
4. decagon D. 6 sides
5. nonagon E. 5 sides


Match each polygon with the correct area formula.

6. triangle A. \( A = lw \)
7. regular pentagon B. \( A = \frac{1}{2}bh \)
8. trapezoid C. \( A = \left( \frac{1}{2} a \right) h \)
9. rectangle D. \( A = \frac{1}{2} (b_1 + b_2)h \)


10. The area of a rectangular field is 6800 square meters. If the width of the field is 80 meters, what is the perimeter of the field? Draw a diagram.

Draw a rectangle. The length of the field is 85 meters because \( \frac{6800}{80} = 85 \).
So, the perimeter of the field is 330 meters because \( 2(80) + 2(85) = 330 \).
11. Find the area of the trapezoid shown.

The area is 96 square centimeters.

12. A side length of the square is 14 inches. The diameter of the circle is 8 inches. Find the area of the shaded portion. Use 3.14 for π.

Find the areas of the square and circle, then find the difference between the areas:

\[ 196 - 16\pi = 145.76 \text{ square inches}. \]

13. The figure shown consists of a rectangle and two congruent circles. If the area of the rectangle is 1250 square feet, what is the radius of one of the circles?

The radius of one of the circles is 12.5 feet.
Post-Test

Name ___________________________________________________ Date _____________________

Match each polygon to the correct number of its sides.

1. octagon  A. 3 sides
2. triangle  B. 11 sides
3. heptagon  C. 4 sides
4. 11-gon    D. 7 sides
5. square    E. 8 sides


Match each polygon with the correct area formula.

6. triangle  A. \( A = lw \)
7. rectangle  B. \( A = \frac{1}{2}bh \)
8. parallelogram  C. \( A = \left( \frac{1}{2}a \right)n \)
9. regular octagon  D. \( A = bh \)


10. The area of a rectangular room is 144 square feet. If the length of the room is 16 feet, what is the perimeter of the room? Draw a diagram.

Draw a rectangle. The width of the room is 9 feet because \( \frac{144}{16} = 9 \).

So, the perimeter is 50 feet because \( 2(16) + 2(9) = 50 \).
11. Find the area of the trapezoid shown.

The area is 222 square yards.

12. The diameter of the large circle is 6 feet. The diameter of the small circle is 2 feet.
Find the area of the shaded portion. Leave your answer in terms of $\pi$.

Find the areas of the small and large circles, then find the difference between the area of the two circles:
$9\pi - \pi = 8\pi$ square feet.

13. The figure shown consists of a rectangle and 10 congruent semicircles. If the perimeter of the rectangle is 88 centimeters, what is the radius of one of the semicircles?

The radius of one of the semicircles is 4.4 centimeters.
1. Find the area of the region bound by the given coordinates on the graph shown.

Connect point (6, 0) with point (13, 14) to form two triangles. The area of the large triangle is \( \frac{1}{2} \times 13 \times 14 = 91 \) square units. The area of the small triangle is \( \frac{1}{2} \times 3 \times 6 = 9 \) square units. The total area of the region is \( 9 + 91 = 100 \) square units.

2. Draw a rectangle that has a perimeter of 36 centimeters and an area of 65 centimeters. Include the length and width on the sketch.

3. Compare the area of \( \triangle BGR \) to \( \triangle BTR \). Which triangle has the greater area? Explain.

The areas are equal because the triangles have the same height and same base.
4. The area of a parallelogram is 85 square meters and its base is 17 meters long. What is the height of the parallelogram?

The height is \( \frac{85}{17} = 5 \) meters.

5. All of the line segments in the figure are either vertical or horizontal. Find the perimeter of the figure.

The perimeter is \( 12 + 12 + 8 + 8 = 40 \) centimeters.

6. All of the line segments in the figures are either vertical or horizontal. Compare the areas and perimeters of the figures. Describe what you find.

Figure 1:

- \( P = 6 + 2 + 4 + 3 + 2 + 5 = 22 \) meters
- \( A = 6(2) + 3(2) = 18 \) square meters

Figure 2:

- \( P = 12 + 4 + 8 + 6 + 4 + 10 = 44 \) meters
- \( A = 12(4) + 6(4) = 72 \) square meters

The perimeter of Figure 2 is twice the perimeter of Figure 1 but the area of Figure 2 is four times the area of Figure 1.
End of Chapter Test

Name ___________________________________________________ Date _____________________

1. The area of trapezoid $JACK$ is 210 square inches. The length of $\overline{AB}$ is 10 inches and the length of $\overline{KC}$ is 24 inches. What is the length of $\overline{JA}$?

![Trapezoid Diagram]

18 inches

2. What is the approximate area of the annulus (shaded area)? Use 3.14 for $\pi$.

![Annulus Diagram]

122.46 square centimeters
Find the area of the region bounded by the line segments in Questions 3 and 4.

3. (4, 17) (16, 17)

192 square units

4. (3, 13) (12, 13)

140.5 square units
5. The area of a parallelogram is 322 square meters and the base is 14 meters long. What is the height of the parallelogram? 23 meters

6. All of the line segments in the figure shown are either vertical or horizontal. What is the perimeter of the figure? 20 inches

7. All of the line segments in the figure shown are either vertical or horizontal. What is the perimeter of the figure? 76 yards

8. All of the line segments in the diagram of the bathroom floor shown are either vertical or horizontal. How many one-inch square tiles would it take to tile the entire floor? 12,528 tiles
9. You are making a black and white checkerboard banner, which will have an area of 1296 square inches. The banner will have eight rows and eight columns of squares, just as a real checkerboard. What is the perimeter of the banner?

\[ s^2 = 1296 \text{ square inches} \]
\[ s = 36 \text{ inches} \]
\[ 4s = 144 \text{ inches} \]

The perimeter is 144 inches.

10. You are making a nylon kite. The kite sticks form right angles with each other, and the longer stick intersects the middle of the shorter stick. If the longer stick is 36 inches and the shorter stick is 22 inches, find the amount of nylon that you need to make the kite.

\[ A = \frac{1}{2} (36)(11) + \frac{1}{2} (36)(11) \]
\[ = 198 + 198 \]
\[ = 396 \text{ square inches} \]

11. You are designing a triangular pennant. You have 400 square inches of fabric to make the pennant. You want the pennant to hang down 50 inches when suspended from its base. If you use all of the fabric to make the pennant, how long will the base be?

\[ 400 = \frac{1}{2} b(50) \]
\[ 400 = 25b \]
\[ 16 = b \]

The base will be 16 inches long.

12. A lawn sprinkler spins around in a circle and has a 6-foot spray. Find the approximate area the lawn sprinkler waters. Use 3.14 for \( \pi \).

\[ A = \pi (6^2) \]
\[ = 36\pi \]
\[ = 113.04 \text{ square feet} \]
Standardized Test Practice

1. What is the name of the polygon shown at the right?
   a. decagon
   b. octagon
   c. hexagon
   d. pentagon

2. What is the area of the region bounded by the five line segments?
   a. 25 square units
   b. 12.5 square units
   c. 20 square units
   d. 15 square units

3. Find the area of the trapezoid.
   a. 47.4 square centimeters
   b. 110.6 square centimeters
   c. 162 square centimeters
   d. 35.8 square centimeters

4. The area of a rectangle is 322 square meters and the length is 14 meters. What is the height of the rectangle?
   a. 23 meters
   b. 48 meters
   c. 12 meters
   d. 4508 meters
5. All of the line segments in the figure at the right are either vertical or horizontal. What can you conclude about the perimeter?
   a. The perimeter of this figure is greater than 30 inches
   b. The perimeter of this figure is less than 30 inches
   c. The perimeter of this figure is equal to than 30 inches
   d. The answer cannot be determined from the given information.

6. All of the line segments in the diagram of the kitchen floor shown at the right are either vertical or horizontal. How many one-inch square tiles would it take to tile the entire floor?
   a. 104 tiles
   b. 504 tiles
   c. 14,976 tiles
   d. 15,840 tiles

7. The vertices of trapezoid PHAT lie on a circle, as shown. Trapezoid PHAT has a height of 12 centimeters and the length of PT is 20 centimeters. If the area of PHAT is 300 square centimeters, what is the approximate area of the shaded region?
   a. 407 square centimeters
   b. 1242 square centimeters
   c. 2526 square centimeters
   d. 25,134 square centimeters
8. The vertices of a regular pentagon lie on a circle. The pentagon has an area of 6552 square units and a perimeter of 312 units. What is the length of the apothem?
   a. 10.5 units  
   b. 21 units  
   c. 42 units  
   d. 49.6 units

9. The area of trapezoid $JACK$ is 34 square yards. The length of $AB$ is 4 yards and the length of $KC$ is 9.6 yards. What is the length of $JA$?
   a. 4.8 yards  
   b. 5.6 yards  
   c. 8.5 yards  
   d. 7.4 yards

10. Trapezoid $YBAR$ has an area of 84 square feet. Its height is 8 feet and the length of $BA$ is 6 feet. What is the area of the shaded region?
    a. 24 square feet  
    b. 48 square feet  
    c. 51 square feet  
    d. 60 square feet

11. The diameter of the large circle is 54 inches. The diameter of the small circle is 18 inches. Calculate the area of the shaded portion.
    a. $18\pi$ square inches  
    b. $36\pi$ square inches  
    c. $648\pi$ square inches  
    d. $2592\pi$ square inches
12. The height of the figure shown at the right is 20 inches, and the width of the figure is 10 inches. All the line segments are either vertical or horizontal. Calculate the perimeter of this figure.

a. 30 inches
b. 50 inches
c. 60 inches
d. Cannot be determined from the given information.
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